AD-762 148

CHAPMAN-KIRK REDUCTION OF FREE-FLIGHT RANGE DATA TO OBTAIN NONLINEAR AERODYNAMIC COEFFICIENTS

Robert H. Whyte, et al

Ballistic Research Laboratories
Aberdeen Proving Ground, Maryland

May 1973

DISTRIBUTED BY:

NTIS

National Technical Information Service
U. S. DEPARTMENT OF COMMERCE
5285 Port Royal Road, Springfield Va. 22151

AD 762148

BRL MR 2298

BRL

AD

MEMORANDUM REPORT NO. 2298

CHAPMAN-KIRK REDUCTION OF FREE-FLIGHT RANGE DATA TO OBTAIN NONLINEAR AERODYNAMIC COEFFICIENTS

by

Robert H. Whyte
Angela Jeung
James W. Bradley

May 1973

Approved for public release; distribution unlimited.

USA BALLISTIC RESEARCH LABORATORIES
ABERDEEN PROVING GROUND, MARYLAND

58

White Section ☒
Buff Section ☐

BY

DISTRIBUTION/AVAILABILITY CODES

Dist.	AVAIL. and/or SPECIAL	
A		

UNCLASSIFIED
Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author)
U. S. Army Ballistic Research Laboratories
Aberdeen Proving Ground, Maryland 21005

2a. REPORT SECURITY CLASSIFICATION
Unclassified

2b. GROUP

3. REPORT TITLE
CHAPMAN-KIRK REDUCTION OF FREE-FLIGHT RANGE DATA TO OBTAIN NONLINEAR AERODYNAMIC COEFFICIENTS

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

5. AUTHOR(S) (First name, middle initial, last name)
Robert H. Whyte
Angela Jeung
James W. Bradley

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
MAY 1973	~~47~~ 55	19

8a. CONTRACT OR GRANT NO.

b. PROJECT NO. RDT&E 1T061102A33D

c.

d.

9a. ORIGINATOR'S REPORT NUMBER(S)
BRL MEMORANDUM REPORT NO. 2298

9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. DISTRIBUTION STATEMENT
Approved for public release; distribution unlimited.

11. SUPPLEMENTARY NOTES

12. SPONSORING MILITARY ACTIVITY
U. S. Army Materiel Command
Washington, D. C. 20315

13. ABSTRACT

The Chapman-Kirk technique for obtaining the parameters in a system of differential equations was applied to time, position and orientation measurements taken along the trajectories of twelve rounds fired in the BRL Transonic Range. The twelve rounds represent four spin-stabilized projectile types: the M71, the M329A1 with and without extension and the M329A1E1. The rounds were previously reduced at BRL by standard, linear reduction techniques; the Chapman-Kirk reduction was carried out by the General Electric Company under contract to BRL. A comparison of the BRL and GE results shows good agreement in the linear coefficients. The Chapman-Kirk technique has the advantage - particularly for large yaw rounds, where the linear analysis breaks down - that it can determine from a single trajectory the values of the nonlinear coefficients present in the equations of motion.

DD FORM 1 NOV 65 1473 REPLACES DD FORM 1473, 1 JAN 64, WHICH IS OBSOLETE FOR ARMY USE.

UNCLASSIFIED
Security Classification

1a

B A L L I S T I C   R E S E A R C H   L A B O R A T O R I E S

MEMORANDUM REPORT NO. 2298

RHWhyte/AJeung/JWBradley/ds
Aberdeen Proving Ground, Md.
May 1973

CHAPMAN-KIRK REDUCTION OF FREE-FLIGHT RANGE DATA TO OBTAIN NONLINEAR AERODYNAMIC COEFFICIENTS

ABSTRACT

The Chapman-Kirk technique for obtaining the parameters in a system of differential equations was applied to time, position and orientation measurements taken along the trajectories of twelve rounds fired in the BRL Transonic Range. The twelve rounds represent four spin-stabilized projectile types: the M71, the M329A1 with and without extension and the M329A1E1. The rounds were previously reduced at BRL by standard, linear reduction techniques; the Chapman-Kirk reduction was carried out by the General Electric Company under contract to BRL. A comparison of the BRL and GE results shows good agreement in the linear coefficients. The Chapman-Kirk technique has the advantage - particularly for large yaw rounds, where the linear analysis breaks down - that it can determine from a single trajectory the values of the nonlinear coefficients present in the equations of motion.

TABLE OF CONTENTS

Preceding page blank

LIST OF TABLES

Preceding page blank

LIST OF SYMBOLS

$\vec{A}$	vector of the N23 parameters a_n to be determined
A_1	$\frac{\pi \rho d^2}{8m}$ $[L^{-1}]$
A_2	$\frac{\pi \rho d^3}{8 I_y}$ $[L^{-2}]$
A_3	$\frac{\pi \rho d^4}{8 I_x}$ C_{ℓ_p} $[L^{-1}]$
a_n	the n-th unknown constant parameter to be determined (n = 1, 2, . . . N23)
a_R	$\tan^{-1} (\dot{y}/\dot{x})$, see Fig. 2c
a_1, a_2, a_3	coefficients in an expansion of time as a cubic in distance, eq. (36)
$\vec{B}$	vector from the range system origin in the direction of the positive X_1-axis, see Fig. 2a
$\vec{C}$	Coriolis acceleration, eq. (64a,b)
C_A	$A_1 \bar{C}_D V_o$ $[T^{-1}]$, eq. (38)
C_D	drag coefficient, $\frac{\lvert \text{drag force} \rvert}{m A_1 V^2}$, eq. (30)
C_{Dj}	coefficient of δ^j (j = 0, 2) in the expansion of C_D
C_{DA}	axial drag coefficient, $(1 - \delta^2) C_D$
C_{DAj}	coefficient of δ^j (j = 0, 2) in the expansion of C_{DA}
C_{DR}	BRL range value of C_D, eq. (36)
$\bar{C}_D$	average value of C_D, eq. (35)
C_{ℓ_p}	roll damping moment coefficient, $\pm \frac{\lvert \text{roll damping moment} \rvert}{m d^2 A_1 V p}$, eq. (47)

Preceding page blank

LIST OF SYMBOLS (Continued)

$C_{M_{p\alpha}}$	Magnus moment coefficient, $\pm \dfrac{\lvert\text{Magnus moment}\rvert}{m\, d^2\, A_1\, V\, p\, \delta}$, eq. (51)
$C_{M_{p\alpha j}}$	coefficient of δ^j ($j = 0, 1, 2, 4, 6, 8$) in the expansion of $C_{M_{p\alpha}}$
C_{M_q}	damping moment coefficient, $\pm \dfrac{\lvert\text{damping moment}\rvert}{m\, d^2\, A_1\, V\, \lvert\dot\theta + i\, \dot\psi \cos\theta\rvert}$, eq. (51)
$C_{M_{qj}}$	coefficient of δ^j ($j = 0, 2$) in the expansion of C_{M_q}
C_{M_α}	static moment coefficient, $\pm \dfrac{\lvert\text{static moment}\rvert}{m\, d\, A_1\, V^2\, \delta}$, eq. (51)
$C_{M_{\alpha j}}$	coefficient of δ^j ($j = 0, 2, 4$) in the expansion of C_{M_α}
$C_{N_{p\alpha}}$	Magnus force coefficient, $\pm \dfrac{\lvert\text{Magnus force}\rvert}{m\, d\, A_1\, V\, p\, \delta}$, eqs. (59, 66)
$C_{N_{p\alpha j}}$	coefficient of δ^j ($j = 0, 2$) in the expansion of $C_{N_{p\alpha}}$
C_{N_α}	normal force coefficient, $\pm \dfrac{\lvert\text{normal force}\rvert}{m\, A_1\, V^2\, \delta}$, eqs. (59, 66)
$C_{N_{\alpha j}}$	coefficient of δ^j ($j = 0, 2$) in the expansion of C_{N_α}
$\vec{C}_T$	centripetal acceleration, eq. (63), [LT^{-2}]
$C_{Y_{p\alpha}}$	$2\, C_{N_{p\alpha}}$, the Magnus force coefficient of Reference 14
D	N1 x N4 matrix of measurements d_{im}
$\vec{D}$	drag force, eq. (30), [MLT^{-2}]
d	reference diameter, [L]
d_{im}	the measured value of y_i at $x = x_m$

LIST OF SYMBOLS (Continued)

E_R	$\sin^{-1}(\dot{z}/V)$, see Fig. 2c
$\vec{F}$	sum of the aerodynamic forces acting on the projectile, $[MLT^{-2}]$
F_X, F_Y, F_Z	range components of $\vec{F}$
F_1, F_2, F_3	fixed-plane components of $\vec{F}$
$f_i(\ )$	single-valued elementary function
$\vec{G}$	gravitational force, eq. (21), $[MLT^{-2}]$
g	magnitude of the gravitational acceleration, $[LT^{-2}]$
I_x, I_y	axial and transverse moments of inertia, $[ML^2]$
$\vec{L}$	angular momentum vector, $[ML^2T^{-1}]$
L_1, L_2, L_3	fixed-plane components of $\vec{L}$, eq. (42)
$\vec{M}$	sum of the aerodynamic moments acting on the projectile, $[ML^2T^{-2}]$
M_1, M_2, M_3	fixed-plane components of $\vec{M}$, eq. (43)
m	mass, [M]
N1	the number of measured dependent variables in a system of equations
N2	the total number of dependent variables in a system of differential equations
N3	the number of unknown parameters appearing explicitly in a system of differential equations
N4	the number of measurements taken on each of the N1 dependent variables
N23	N2 + N3, the number of unknown constants in a system of equations
n	rifling, [cal/rev], Table I

LIST OF SYMBOLS (Continued)

PE — probable error

P_{in} — $\frac{\partial y_i}{\partial a_n}$, the influence (or sensitivity) coefficients, eq. (3)

p — the X_1-component of the angular velocity of the missile-fixed system with respect to the range system, eqs. (40-41), [rad/sec]

$\vec{R}$ — position vector of the missile's CG, [L]

r — arclength along the trajectory of the missile's CG, [M]

t — time

t* — value of time at x = x*

u_1, u_2, u_3 — fixed-plane components of $\vec{V}$, $[LT^{-1}]$

$\vec{V}$ — velocity vector, $[LT^{-1}]$

V — magnitude of $\vec{V}$, $[LT^{-1}]$

$\vec{X}$ — vector of the N4 values x_m of the independent variable at which measurements are taken

XYZ — the axes of a range (earth-fixed) coordinate system; the X-axis is directed down-range along the intersection of a horizontal plane with the vertical plane containing the gun; the Y-axis lies in this horizontal plane, directed to the left of an observer facing downrange; the Z-axis is directed upward.

$X_1X_2X_3$ — the axes of a fixed-plane coordinate system; the X_1-axis lies along the missile's longitudinal axis, directed from the CG (origin) to the nose; the X_2-axis is constrained to lie in the horizontal plane, directed to the left of an observer facing in the direction of the positive X_1-axis; the X_3-axis is directed upward.

$X_1X_2'X_3'$ — the axes of the fixed-plane system of References 1 and 2, where $X_2' = -X_2$, $X_3' = -X_3$.

x	the independent variable of a system of equations
x, y, z	range components of $\vec{R}$
$\dot{x}, \dot{y}, \dot{z}$	range components of $\vec{V}$
x_m	the value of the independent variable x at which the m-th measurement is taken, m = 1, 2, . . . N4
x^*	the mid-range value of the down-range coordinate x
y_j	the dependent variables of a system of equations
α_{nk}	the coefficients of the increments Δa_k in the differential corrections normal equations, see eqs. (6-7)
$\tilde{\alpha}_{nk}$	the (n, k)-th element of the inverse of matrix (α_{nk})
α_T	the yaw angle, the total angle of attack, measured (in the $X_1 X_2 X_3$ system) from the velocity vector to the X_1-axis.
β_n	the terms on the right-hand side of the differential corrections normal equations, see eqs. (6, 8)
γ_{kn}	a possible replacement for α_{nk}, suggested by Marquardt, eq. (15)
Δa_k	the increments by which the parameters are changed in the differential corrections process
δ	sine of the yaw angle, eq. (17)
ε	the sum of the squares of the residuals in a least squares fit, eq. (2)
θ	an Euler angle; see Fig. 2a
θ_A	azimuth of the line-of-fire, that is, the angle measured clockwise from North to the down-range axis (192° for the BRL Transonic Range)
θ_L	latitude of the range, considered positive in the Northern hemisphere (39° 26' at BRL)
θ_m	the missile's angle of yaw; see Fig. 2a and eq. (26)

LIST OF SYMBOLS (Continued)

λ	a constant in Marquardt's scheme, eq. (15)
ξ_2, ξ_3	$-u_2/V$, $-u_3/V$
ρ	air density, considered constant, $[ML^{-3}]$
$\vec{\omega}_E$	the earth's angular velocity vector, eq. (64a)
ω_E	magnitude of $\vec{\omega}_E$, 0.00007292 rad/sec
$\vec{\omega}_{S_1(S_2)}$	(where S_1 and S_2 can be FP, R or R′) angular velocity of the S_1 coordinate system with respect to the S_2 coordinate system

Subscripts:

$(\ )_{FP}$	components in a fixed-plane $(X_1 X_2 X_3)$ system
$(\ )_R$	components in a range (XYZ) system
$(\ )_{R'}$	components in an intermediate $(X'Y'Z')$ system
$\left.\dfrac{d^2\vec{R}}{dt^2}\right\vert_I$	second derivative of $\vec{R}$ in an inertial system
$(\ )_o$	initial condition, taken here to be the value at the first data station

I. INTRODUCTION

In free flight studies, the main task is to determine from observations of a missile's motion the values of the aerodynamic coefficients appearing in the differential equations describing that motion. For normal enclosed-range firing conditions (symmetric shell, nearly horizontal flight, small yaw, constant or only slowly varying spin, etc.), we can approximate the solution to the differential equations quite adequately by convenient closed-form expressions. These expressions involve certain constants directly related to the aerodynamic coefficients. The values of these constants are determined by a least squares fit of the observed data to the closed-form expressions.

For the past two decades, the general technique of fitting observed data to convenient closed-form expressions has been the heart of free-flight, enclosed-range data reduction[1,2*]. Highly successful results have been obtained for a variety of missile shapes and sizes. Over the years, the technique has been gradually refined and extended to cover many types of force and moment nonlinearities. Unfortunately, such extension often requires that a number of rounds be fired at different Mach numbers, yaw levels, and so on, to obtain a single set of coefficients. The process can be costly in dollars and time. Moreover, the technique can be stretched only so far; an occasional round has defied analysis by conventional procedures.

It is relatively easy in these troublesome nonlinear situations to specify - by experience and by cunning - the sort of nonlinear terms that must appear in the differential equations of motion in order to produce the observed behavior. It is much more difficult to find convenient pseudo-solutions that will (1) represent the motion adequately and (2) contain constants that can be easily related to the coefficients of the differential equations. What we need in these situations is a method that doesn't require any knowledge or assumptions on our part regarding the form of the solution to the differential equation.

The problem is essentially one of parameter optimization. We are given

a. the form of a set of differential equations involving unknown constant parameters (coefficients and initial conditions);

b. a set of first estimates for the parameters;

c. a set of discrete measurements on one or more of the dependent variables;

d. a criterion function of the parameters (say, the sum of the squares of the residuals for a given fit).

References are listed on page 59.

The problem is to devise a routine for adjusting the parameters automatically so as to minimize the criterion function.

The problem has received much attention, particularly in recent years with the proliferation of high-speed computers, and many ingenious schemes for optimizing the parameters have been proposed. For continuous rather than discrete input data, Meissinger[3,4] approached the criterion function minimum by a path of approximately steepest descent, using an analog computer. On the other hand, the techniques devised by Goodman[5-7] and by the team of Chapman and Kirk[8] to handle discrete data are better suited to the digital computer. In this paper, we will be concerned primarily with the Chapman-Kirk technique. The Goodman and Chapman-Kirk methods are quite similar, although Chapman and Kirk were unaware of Goodman's work when they presented their own results at the AIAA Seventh Aerospace Sciences Meeting, New York, 1969. From the standpoint of the aerodynamicist, Chapman and Kirk's contribution was to apply the process successfully to representative aerodynamic cases and, perhaps more important, to present their results at the meeting and later in a journal[8] where it came to aerodynamicists' attention. (It is unfortunate but often true that when a pertinent article such as Goodman's appears in a mathematical journal, the aerodynamicist either overlooks it or fails to recognize its applicability to his work.)

Applications[9-13] of the Chapman-Kirk technique are growing more and more sophisticated. The present report documents one such application carried out by the Armament Department, General Electric Company, Burlington, Vermont, for the U. S. Army Aberdeen Research and Development Center (ARDC), Aberdeen Proving Ground, Maryland, under Government Contract No. DAAD05-71-C-0265 during the period 4 February to 20 April 1971. The results of this work have also been issued as a General Electric report[14].

The raw data for this study consisted of time, position and orientation measurements at discrete points along the trajectory for twelve rounds fired in the Transonic Free Flight Range[15], Ballistic Research Laboratories (BRL). The twelve rounds consisted of four spin-stabilized projectile types (see Table I and Figure 1). Each of the rounds had been previously reduced[11,16] by the usual range technique (with varying degrees of success) and each was hand-picked for the present assignment.

Some of the twelve rounds could be fitted by assuming relatively simple force and moment systems; these rounds were chosen to enable the Chapman-Kirk technique to get a foot in the door. Other rounds of the twelve were oddities that had already annoyed and frustrated a team of data analysts; these rounds were chosen to give the Chapman-Kirk technique a good work-out. The primary purpose of the present study was to see how well the Chapman-Kirk technique could determine the values of the nonlinear coefficients present in the force and moment expressions.

II. THE CHAPMAN-KIRK TECHNIQUE

Suppose for the moment that we have a system defined by elementary equations*. Let

N1 = the number of measured dependent variables

N23 = the number of unknown constants in the system

N4 = the number of measurements taken on each of the N1 dependent variables

where N4 is greater than N23. (The notation here is not entirely capricious. An N2 and N3, lying between N1 and N4, will be introduced later and N23 will be the sum of N2 and N3.) Assume that we can write our system of equations in the form

$$y_i = f_i (x, a_1, a_2, \ldots, a_{N23}), \ i = 1, 2, \ldots, N1 \qquad (1)$$

where y_i is the i-th dependent variable (i = 1, 2, . . . N1)

f_i is a single-valued elementary function

x is the independent variable

a_n is the n-th unknown constant parameter (n = 1, 2, . . ., N23)

We are given a set of measurements, which we represent by the N1 x N4 matrix $D = (d_{im})$, and a vector $\vec{X} = (x_m)$ of the corresponding N4 values of the independent variable. That is,

d_{im} = the measured value of y_i at x_m

i = 1, 2, . . ., N1

m = 1, 2, . . ., N4

For any parameter vector $\vec{A} = (a_n)$, we can obtain from (1) the corresponding solution $y_i (x_m)$ at each point x_m. The problem is to determine the value of $\vec{A}$ that minimizes ε, the sum of the squares of the residuals:

**By an elementary equation, we mean an equation involving only elementary functions and a finite number of arithmetical operations. Specifically, we are excluding differential equations.*

$$\varepsilon = \sum_{m=1}^{N4} \sum_{i=1}^{N1} [d_{im} - y_i (x_m)]^2 \tag{2}$$

(For convenience, we omit weighting factors in (2) and the succeeding discussion; such factors might be needed, for example, to insure that the terms in (2) are dimensionally equal.) Now ε will be at a minimum only when its partial derivatives with respect to each of the parameters is zero. We introduce a convenient notation for these partial derivatives:

$$P_{in} = \frac{\partial y_i}{\partial a_n} \tag{3}$$

$$i = 1, 2, \ldots, N1$$

$$n = 1, 2, \ldots, N23$$

Because they reflect the influence of each parameter change, the P_{in} are sometimes called "influence" (or "sensitivity") coefficients. For a minimum ε, we must have

$$\frac{\partial \varepsilon}{\partial a_n} = -2 \sum_{m=1}^{N4} \sum_{i=1}^{N1} [d_{im} - y_i (x_m)] P_{in} = 0 \tag{4}$$

Equations (4) constitute a set of N23 equations in the N23 unknown parameters; the values of the parameters satisfying (4) are the desired optimum values.

If the functions f_i are <u>linear</u> in the parameters a_n:

$$f_i = a_1 \phi_{i_1} (x) + a_2 \phi_{i_2} (x) + \ldots + a_{N23} \phi_{iN23} (x)$$

then

$$P_{in} = \phi_{in} (x)$$

and set (4) is also linear in the parameters; hence (4) is easily solvable (in theory). If the functions f_i are <u>nonlinear</u> in the parameters, however, then solving (4) can be quite difficult. The usual way out of this difficulty is to approximate the variables y_i by their

linearly truncated Taylor expansions about a given set of values for the parameters:

$$y_i \cong \hat{y}_i + \sum_{k=1}^{N23} \hat{P}_{ik} \, \Delta a_k \qquad (5)$$

where the circumflex (^) denotes evaluation at the given parameter values and where Δa_k is an indicated change in the given value of a_k.

If (5) is substituted in (4), and if the P_{in} in (4) are replaced by $\hat{P}_{in}$, we have

$$\sum_{m=1}^{N4} \sum_{i=1}^{N1} \left[d_{im} - \hat{y}_i - \sum_{k=1}^{N23} \hat{P}_{ik} \, \Delta a_k \right] \hat{P}_{in} = 0$$

or

$$\sum_{k=1}^{N23} \alpha_{nk} \, \Delta a_k = \beta_n, \; n = 1, 2, \ldots, N23 \qquad (6)$$

where

$$\alpha_{nk} = \sum_{m=1}^{N4} \sum_{i=1}^{N1} \hat{P}_{in} (x_m) \, \hat{P}_{ik} (x_m) \qquad (7)$$

$$\beta_n = \sum_{m=1}^{N4} \sum_{i=1}^{N1} \left[d_{im} - \hat{y}_i (x_m) \right] \hat{P}_{in} (x_m) \qquad (8)$$

Equation (6) represents a set of N23 linear equations in the N23 increments Δa_n. Hence we can solve (6) for the increments:

$$\Delta a_n = \sum_{k=1}^{N23} \tilde{\alpha}_{nk} \, \beta_k, \; n = 1, 2, \ldots, N23 \qquad (9)$$

where

$$\tilde{a}_{nk} = \text{the } (n, k)\text{-th element of the inverse of matrix } (\alpha_{nk})$$

$$= \frac{\text{cofactor of element } \alpha_{nk}}{\text{determinant of matrix } (\alpha_{nk})}$$

The new value of a_n is then obtained by adding Δa_n to the old value.

If the initial estimates of the parameters are "close enough" to the "true" values, (5) will be an adequate approximation and the new values of the parameters will produce a smaller criterion function ε. The process can then be repeated as many times as necessary until some convergence criterion is satisfied. The probable error of the fit at the end of any iteration is given by

$$PE = 0.6745 \left[\frac{\varepsilon}{N1 \times N4 - N23}\right]^{\frac{1}{2}} \tag{10}$$

and an estimate of the probable error in a_n is given by

$$E\,(a_n) = (\tilde{a}_{nn})^{\frac{1}{2}} \cdot PE \tag{11}$$

This, in brief, is the well-known process of "differential corrections," as applied to elementary equations that are nonlinear in the parameters.

For a system of ordinary differential equations, the situation is naturally more complicated. Assume that the given system of differential equations has been reduced to a system of N2 first order, possibly non-linear equations*:

$$\left.\begin{aligned} \frac{dy_j}{dx} &= f_j\,(x, y_1, y_2, \ldots, y_{N2}, a_1, a_2, \ldots, a_{N3}) \\ y_j\,(x_o) &= a_{N3+j} \\ j &= 1, 2, \ldots, N2 \end{aligned}\right\} \tag{12}$$

It is not necessary when carrying out the Chapman-Kirk technique to reduce the given system to first order equations; this was done here merely to aid the exposition.

where

$N2$ = the total number of dependent variables in the system

$N3$ = the number of unknown parameters appearing explicitly in the differential equations

and where N1 and N4 are defined as before. Note that we have juggled the notation so that while only N3 parameters appear explicitly in (12), there are still N23 (= N2 + N3) parameters to be determined:

$a_1, a_2, \ldots, a_{N3}$ (the N3 explicit parameters)

$a_{N3+1}, a_{N3+2}, \ldots, a_{N23}$ (the N2 unknown initial conditions)

where

$$1 \leq N1 \leq N2 \leq N23 < N4 \tag{13}$$

The same differential correction technique that was applied to (1) can be applied to (12). Equations (6-11), which depend only on the definition (2) of the criterion function ε, are still valid. However, a new difficulty arises when the given set of equations are ordinary differential equations: how do we evaluate the dependent variables and their partial derivatives appearing in (7) and (8)? For the case of non-differential equations, this was no problem; we were presumably given explicit expressions for each of the variables and could easily write down expressions for the required partials. For a given set of differential equations, however, we will not, in general, know the form of the solution. Values of the dependent variables can be obtained by some numerical integration scheme, but part of the problem remains: how do we obtain the required values of the partial derivatives?

Chapman and Kirk tried various schemes for evaluating these partial derivatives and finally settled on the method* of "parametric differentiation." This method consists of formally differentiating the given set (12) of differential equations with respect to each of the N23 constants to be determined. We have

$$\frac{\partial}{\partial a_n}\left(\frac{dy_j}{dx}\right) = \frac{\partial f_j}{\partial a_n}$$

**The method is not new. It was used, for example, by the previously cited Meissinger and Goodman (and apparently by Knalder[17], with whose work we are unfamiliar but who is referenced in References 10 and 12).*

or, assuming that the y_j are continuous in x and a_n, so that the order of differentiation can be interchanged,

$$\frac{d\,P_{jn}}{dx} = \frac{\partial\, f_j}{\partial\, a_n} \qquad \begin{array}{l} j = 1, 2, \ldots, N2 \\ n = 1, 2, \ldots, N23 \end{array} \qquad (14a)$$

where

$$\begin{aligned} P_{jn}\,(x_o) &= 1 \qquad (\text{if } n - j = N3) \\ &= 0 \qquad (\text{otherwise}) \end{aligned} \qquad (14b)$$

The somewhat strange-looking initial conditions (14b) merely reflect the fact that P_{jn} is initially 1 if and only if a_n represents the initial value of y_j. By (12), this occurs if and only if $a_n = a_{N3} + j$; hence if and only if $n = N3 + j$.

What we have done above is to derive an auxiliary set of N2 x N3 equations (14) whose solutions are the partials P_{jn}, some of which (those for $j < N1$) are needed in solving (6). For a given set of estimates of the parameters and initial conditions, we can integrate by some numerical scheme both the original set of equations (12) and the auxiliary set (14). (The original set may or may not be linear; the auxiliary set will always be linear.) The numerical integration yields the values of the dependent variables and the influence coefficients required to solve (6) for the parameter changes.

Except for this more laborious way of determining $y_i\,(x_m)$ and $P_{in}\,(x_m)$, the procedure for determining the unknown constants of a set of differential equations is the same as for non-differential equations.

As a final aside, we note a possible future improvement. In the N23-dimensional parameter space, the truncated Taylor series technique proceeds from a given point (whose coordinates are the given estimates of the N23 constants) in the direction of the vector $\Delta A = (\Delta a_n)$ obtained by solving (6). The method of steepest descent, on the other hand, proceeds from the given point in the direction of the negative gradient of ε. Marquardt[18,19] points out that these two directions are nearly perpendicular, while the optimum direction lies somewhere in between. To proceed in approximately the optimum direction, he suggests replacing α_{kn} in (6) by

$$\gamma_{kn} = \begin{cases} (1 + \lambda)\ \alpha_{kn} \text{ for } k = n \\ \alpha_{kn} \text{ for } k \neq n \end{cases} \tag{15}$$

where λ is a fudge factor constant whose value should be changed from one iteration to the next according to a few simple rules (the rules are listed - in slightly different form - in References 18 and 19). Using Marquardt's magic λ, the Chapman-Kirk process often converges for initial guesses far outside the previous region of convergence. The Marquard algorithm was pointed out to us by Chapman himself, who has used it (subsequent to the work reported on in Reference 8) with great success in hitherto intractable cases. The algorithm was not used in the investigation covered by this report because we were not aware of it at the time.

III. INPUT DATA, COORDINATE SYSTEMS AND YAW VARIABLES

A missile fired in the BRL Transonic Range is observed at twenty-five spark-photography stations distributed along a 680-foot portion of the trajectory. The observed data at each station consists of

a. the elapsed time t, reckoned from the instant the spark at the first station was triggered by the passing missile. The time error in a properly functioning timer is estimated to be no more than one microsecond. Only about two-thirds of the stations are instrumented at present to furnish timing data.

b. $(x,y,z)_R$: the position vector of the missile's CG in a range (earth-fixed) coordinate system XYZ. The X-axis is directed down-range along the intersection of a horizontal plane with the vertical plane containing the gun; the Y-axis lies in this horizontal plane, directed to the left of an observer facing downrange; the Z-axis is directed upward. The error in any position measurement should be no greater than 0.003 meter.

c. $(0, \xi_2, \xi_3)_{FP}$: the yaw vector in a fixed-plane coordinate system $X_1X_2X_3$. The X_1-axis lies along the missile's longitudinal axis, directed from the CG (the origin of the fixed-plane system) to the nose; the X_2-axis is constrained to lie in the horizontal plane, directed to the left of an observer facing in the direction of the positive X_1-axis; the X_3-axis is directed according to the right-hand rule (that is, upward). If u_1, u_2, u_3 are the velocity components in the fixed-plane system, then

$$\xi_2 = -\frac{u_2}{V}, \quad \xi_3 = -\frac{u_3}{V} \qquad (16)$$

where V is the magnitude of the velocity vector. The minus signs in (16) appear because in range studies the yaw angle is measured from the velocity vector to the X_1-axis*. The magnitude δ of the yaw is given by

$$\delta = \left(\xi_2^2 + \xi_3^2\right)^{\frac{1}{2}} = \frac{\left(u_2^2 + u_3^2\right)^{\frac{1}{2}}}{V} = \sin \alpha_T \qquad (17)$$

where α_T is the yaw angle, the so-called total angle of attack. The angular measurements that yield the yaw components are usually accurate to within 0.002 radian.

Although differential equations describing the yawing motion can be written in terms of ξ_2 and ξ_3, we found it more convenient to work with the related Euler angles ψ and θ. These angles appear in the transformation matrix that converts from range to fixed-plane coordinates. To derive this matrix, assume the existence of a vector $\vec{B}$ extending from the range system origin in the direction of the positive X_1-axis (see Figure 2a). Rotate the range system XYZ about the Z-axis by the angle ψ so that X' (the rotated X-axis) coincides with the projection of the vector $\vec{B}$ on the XY-plane. The magnitude of ψ is not to exceed 180°; if this requires a counterclockwise rotation about the Z-axis (as in Figure 2a), ψ is considered positive and if clockwise, then ψ is considered negative. Then we have

---

**In References 1 and 2, a fixed-plane system $X_1X'_2X'_3$ is used, where the X'_2- and X'_3-axes have opposite directions to the X_2- and X_3-axes, respectively. In this $X_1X'_2X'_3$ system, the yaw angle is measured from the X_1-axis to the velocity vector. If u_1, u'_2, u'_3 are the velocity components and $0, \xi'_2, \xi'_3$ are the yaw components in the $X_1X'_2X'_3$ system, then*

$$\xi'_2 = \frac{u'_2}{V} = -\frac{u_2}{V} = \xi_2$$

and similarly, $\xi'_3 = \xi_3$. That is, the yaw components in the two fixed-plane systems are identical.

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}_{R'} = \begin{pmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}_R \tag{18}$$

Next, rotate the intermediate system $X'Y'Z'$ about the Y'-axis so that X'' (the rotated X-axis) coincides with vector $\vec{B}$. Let θ denote the angle from Z' to $\vec{B}$, where $|\theta| < 90°$. If the rotation is counterclockwise about the Y'-axis, θ is considered positive; if the rotation is clockwise (as in Figure 2a), θ is negative. Then we have

$$\begin{pmatrix} x'' \\ y'' \\ z'' \end{pmatrix}_{FP} = \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix} \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}_{R'} \tag{19}$$

This final system $X''Y''Z''$ has the orientation of the fixed-plane system $X_1 X_2 X_3$. Thus the transformation matrix from range to fixed-plane coordinates is the product of the θ-matrix and the ψ-matrix:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}_{FP} = \begin{pmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ -\sin\psi & \cos\psi & 0 \\ \sin\theta\cos\psi & \sin\theta\sin\psi & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}_R \tag{20}$$

For an assumed flat, nonrotating earth, the gravitational force $\vec{G}$ has the form

$$\vec{G} = (0,\ 0,\ -mg)_R \tag{21a}$$

where the gravitational acceleration g is constant for range firings. Substituting the right-hand side of (21a) in (20), we see that the fixed-plane components of $\vec{G}$ depend on the Euler angle θ:

$$\vec{G} = (mg\sin\theta,\ 0,\ -mg\cos\theta)_{FP} \tag{21b}$$

Note that by the definitions of the two Euler angles, the angular velocity of the intermediate R' $(X'Y'Z')$ system with respect to the range system is

$$\vec{\omega}_{R'(R)} = (0,\ \dot{\theta},\ \dot{\psi})_{R'}$$

Substituting this vector in (19), we obtain the angular velocity of the fixed-plane system with respect to the range system

$$\vec{\omega}_{FP(R)} = (- \dot{\psi} \sin \theta,\ \dot{\theta},\ \dot{\psi} \cos \theta)_{FP} \tag{22}$$

The above discussion gives us the physical interpretation of ψ and θ, but in order to work with these angles, we needed explicit equations relating ψ and θ to the given yaw components ξ_2 and ξ_3. By some elementary but cumbersome vector analysis, it can be shown that the desired relations are

$$\psi = \sin^{-1} \left(\frac{\sin \psi_M}{\cos E_R} \right) + a_R \tag{23}$$

$$\theta = \theta_M - \sin^{-1} \left(\frac{\sin E_R}{\cos \psi_M} \right) \tag{24}$$

where ψ_M and θ_M are the missile's pitch and yaw angles, respectively (see Figure 2b):

$$\psi_M = \sin^{-1} \left(- \frac{u_2}{V} \right) = \sin^{-1} (\xi_2) \tag{25}$$

$$\theta_M = \tan^{-1} \left(\frac{u_3}{u_1} \right) = - \sin^{-1} \left(\frac{\xi_3}{\cos \psi_M} \right) \tag{26}$$

where a_R and E_R are the azimuth and elevation angles, respectively, of the velocity vector in the range system (see Figure 2c):

$$a_R = \tan^{-1} \left(\frac{\dot{y}}{\dot{x}} \right) \tag{27}$$

$$E_R = \sin^{-1} \left(\frac{\dot{z}}{V} \right) \tag{28}$$

where $\dot{x}, \dot{y}, \dot{z}$ are the velocity components in the range system.

For the present analysis of range firings, we made the simplifying assumption that the angles a_R and E_R could be ignored. Letting $a_R = E_R = 0$, equations (23) and (24) reduce to

$$\psi = \psi_M \tag{23a}$$

$$\theta = \theta_M \tag{24a}$$

By equations (17), (25) and (26), the magnitude δ of the yaw can be expressed in terms of the pitch and yaw angles:

$$\delta = (\sin^2 \psi_M + \cos^2 \psi_M \sin^2 \theta_M)^{\frac{1}{2}} \tag{29}$$

IV. THE EQUATIONS OF MOTION

In this section, the working forms of the equations of motion are derived (albeit briefly) from the basic expressions for Newton's Second Law. By writing out this derivation, we can point out where and what kinds of assumptions and simplifications were made and thus facilitate future changes.

The interdependence of the various equations and of the three distinct reductions (drag, yaw and CG) are indicated in Figure 3.

A. The Drag Equation

The classic drag equation has the form

$$\vec{D} + \vec{G} = m \left.\frac{d^2\vec{R}}{dt^2}\right|_I \quad [MLT^{-2}] \tag{30}$$

where

$$\vec{D} = \text{drag force}$$

$$= -(m\, A_1\, C_D\, V)\, \vec{V}$$

$$A_1 = \frac{\pi\, \rho\, d^2}{8m} \quad [L^{-1}]$$

$$\vec{G} = \text{gravitational force}$$

$$\vec{R} = \text{position vector of the missile's CG}$$

and where the subscript I denotes vector differentiation in an inertial system. For enclosed-range studies, the density ρ and hence A_1 are known constants. The drag coefficient C_D is in general a function of Mach number and of δ^2. For each of the twelve Transonic Range rounds studied here, however, we could ignore the Mach number variation over the observed trajectory and assume a linear* dependence of C_D on δ^2:

$$C_D = C_{D_0} + C_{D_2} \delta^2 \tag{31}$$

We seek the X-component of (30) in the range system. For the purpose of performing the drag reduction, we can assume that the range system is an inertial system and hence ignore the centripetal and Coriolis accelerations that arise in any earth-fixed system. Then the X-component of (30) can be written as

$$\left.\begin{aligned} \ddot{x} &= - A_1 C_D V \dot{x} \\ &= - A_1 C_{DX} V^2 \end{aligned}\right\} \tag{32}$$

where

$$C_{DX} = \left(\frac{\dot{x}}{V}\right) C_D = C_D \cos E_R \cos a_R$$

$$= \text{down-range drag coefficient}$$

For the present Transonic Range studies, (32) has the disadvantage that the independent variable t - which should be known exactly at each point - is obtained only at about two-thirds of the spark stations (and obtained with sufficient accuracy at a considerably smaller fraction). On the other hand, the down-range coordinate x of the missile's CG is usually known very accurately at each station. Thus a reasonable course of action is to convert the independent variable in (32) from t to x. We have

---

**Provision was made in the coding to handle C_D as a quadratic in δ^2, but the higher-order term was never needed.*

$$\left.\begin{aligned} \frac{dt}{dx} &= \frac{1}{\dot{x}} \\ \frac{d\dot{x}}{dx} &= -A_1 C_D V = -A_1 \left(\frac{C_D{}^2}{C_{DX}}\right) \dot{x} \end{aligned}\right\} \quad (33)$$

The presence in (33) of the velocity V is a nuisance for as yet we have no equation for generating V. For the nearly horizontal flights encountered in ballistic ranges, the distinction between V and $\dot{x}$ (= $V \cos E_R \cos a_R$) and between C_D and C_{DX} can be ignored. Thus we can write (33) in the final form:

$$\left.\begin{aligned} \frac{dt}{dx} &= \frac{1}{V} \\ \frac{dV}{dx} &= -\left(C_{D_o} + C_{D_2} \delta^2\right) A_1 V \end{aligned}\right\} \quad (34)$$

The drag reduction - that is, the application of the Chapman-Kirk technique to (34) - is normally done before the yaw reduction and so at this stage we know the values of δ^2 only at certain discrete points (namely, at each spark station). From these scattered values, we obtained additional input values of δ^2 at selected values of x by interpolation. (After the yaw reduction has been performed, the drag reduction could be re-done, using the fitted values of δ^2 rather than the raw plus interpolated-raw values. For the present study, this wasn't necessary.)

In addition to the yaw data, the required input for a given round consisted of the measured times t, the measured x values and initial estimates of the two explicit parameters $\left(C_{D_o}, C_{D_2}\right)$ and the two initial conditions (t_o, V_o), where we defined initial conditions as the conditions at the first spark station. The output consisted of

a. the least squares values of the explicit parameters and initial conditions;

b. appropriate error estimates in the above;

c. a reliable correlation of time with distance, so that in the remaining equations of motion, time could be used as the independent variable. Of course, it was not absolutely necessary to revert to a

time base. For theoretical work, the arclength r along the trajectory of the missile's CG (or a nondimensional length r/d) is often a more convenient independent variable than either time or down-range distance. The equations of motion (in particular, the yaw equations) assume simpler forms when r is the independent variable. Since $\dot{r} = V$, our approximation $\dot{x} \cong V$ is equivalent to ignoring the distinction between x and r. We could, then, have written all the equations with respect to this convenient length variable which we measure as x but are free to interpret as r. One reason we didn't is that the present study, while interesting in itself, is also regarded as preliminary, getting-our-feet-wet training for more ambitious studies that will require a time-based set of equations. By working with time-based equations in the present study, we could save considerable coding effort. Comprehensive computer programs, capable of handling the larger problems, were able to handle the current problem as a special case.

One additional bit of information can be gleaned from our reduction: a representative C_D value for each round, obtained by replacing δ^2 in (31) by its mean value:

$$\overline{C}_D = C_{D_o} + C_{D_2} \overline{\delta^2} \tag{35}$$

Although a value for $\overline{\delta^2}$ was available for each round from the BRL yaw reduction, the quantities $\overline{\alpha}_T = \sin^{-1}\left(\sqrt{\overline{\delta^2}}\right)$ listed in Tables II - VI were obtained by a slightly different averaging process than used by BRL. Thus these listed values differ slightly (by less than 4%) from the BRL values. Each representative drag coefficient $\overline{C}_D$ can be compared with the range value, C_{DR}, obtained at BRL by fitting time as a cubic in down-range distance x:

$$\left.\begin{aligned} t &= t^* + a_1 (x - x^*) + a_2 (x - x^*)^2 + a_3 (x - x^*)^3 \\ C_{DR} &= \frac{2 a_2}{A_1 a_1} \\ &= \text{range value of } C_D \text{ at } \left[(t, x, V) = (t^*, x^*, 1/a_1)\right] \end{aligned}\right\} \tag{36}$$

where x* is the given mid-range value of x.

As we will see, the variable V appears in the yaw and CG equations and the question arises: how should we generate it there? We could,

of course, store the large number of discrete velocity values available when we integrate the drag equation (34) numerically. This would be both tedious and unnecessary. Instead, we made a reasonable assumption: for purposes of performing the yaw reduction* on the given Transonic Range rounds, V can be represented adequately over the observed trajectory by a <u>known</u> quadratic in time:

$$V = V_0 + V_1 (t - t_0) + V_2 (t - t_0)^2 \tag{37}$$

Values for V_1 and V_2 in (37) can be obtained in various ways. For example, if we replace C_D with $\overline{C}_D$ in the drag equation, the solution can be written at once:

$$\left.\begin{aligned} V &= V_0 \exp\left[- A_1 \overline{C}_D (x - x_0)\right] \\ &= \frac{V_0}{1 + C_A (t - t_0)} \end{aligned}\right\} \tag{38}$$

where

$$C_A = A_1 \overline{C}_D V_0 \; [T^{-1}]$$

with the truncated series expansion:

$$V = V_0 \left[1 - C_A (t - t_0) + C_A^2 (t - t_0)^2\right] \tag{37a}$$

B. <u>The Roll Equation</u>

Consider a missile-fixed coordinate system $X_1 X_4 X_5$, where the X_1-axis lies along the missile's longitudinal axis (as in the $X_1 X_2 X_3$ fixed-plane system) and where the X_4 and X_5 axes are rigidly attached to the missile in a right-handed system. Then the angular velocity of this missile-fixed system relative to the fixed-plane system is given by

$$\vec{\omega}_{MF(FP)} = (\dot{\phi}, 0, 0)_{FP} \tag{39}$$

**The assumption is not necessary for the CG equations where, as we shall see, V can be generated handily.*

where ϕ is the roll angle (the angle between the $X_2 X_3$ axes and the $X_4 X_5$ axes). By (22) and (39), we can write an expression for the angular velocity of the missile-fixed system with respect to the range system:

$$\left.\begin{aligned} \vec{\omega}_{MF(R)} &= \vec{\omega}_{MF(FP)} + \vec{\omega}_{FP(R)} \\ &= (p,\ \dot{\theta},\ \dot{\psi}\cos\theta)_{FP} \end{aligned}\right\} \quad (40)$$

where

$$p = \dot{\phi} - \dot{\psi}\sin\theta \qquad (41)$$

The angular momentum of a missile with rotational symmetry is then

$$\vec{L} \equiv (L_1,\ L_2,\ L_3)_{FP} = (I_x\ p,\ I_y\ \dot{\theta},\ I_y\ \dot{\psi}\cos\theta)_{FP} \quad [ML^2T^{-1}] \qquad (42)$$

The sum of the moments acting on the missile is equal to the time derivative of the angular momentum:

$$\left.\begin{aligned} \vec{M} \equiv (M_1,\ M_2,\ M_3)_{FP} &= \left.\frac{d\vec{L}}{dt}\right|_I \qquad [ML^2T^{-2}] \\ &= [(\dot{L}_1,\ \dot{L}_2,\ \dot{L}_3) + \vec{\omega}_{FP(R)} \times \vec{L}]_{FP} \end{aligned}\right\} \quad (43)$$

where again the range system is considered an inertial system. Hence by (22) and (42),

$$M_1 = I_x\ \dot{p} \qquad (44)$$

$$M_2 = I_y\ \ddot{\theta} + (I_x\ p + I_y\ \dot{\psi}\sin\theta)\ \dot{\psi}\cos\theta \qquad (45)$$

$$M_3 = I_y\ \ddot{\psi}\cos\theta - (I_x\ p + 2\ I_y\ \dot{\psi}\sin\theta)\ \dot{\theta} \qquad (46)$$

Equation (44) is the roll equation; (45) and (46) constitute the yaw equations.

The roll equation does not depend on the other two moment equations and hence can be solved separately. We define the axial moment as

$$M_1 = \left(\rho \frac{V^2}{2}\right) \left(\frac{\pi d^2}{4}\right) (d) \left(\frac{pd}{V}\right) C_{\ell_p} \quad [ML^2T^{-2}] \qquad (47)$$

where C_{ℓ_p} is the roll damping moment coefficient*. Then the roll equation (44) becomes

$$\dot{p} = A_3\, p\, V \qquad [T^{-2}] \qquad (48)$$

where

$$A_3 = \left(\frac{\pi \rho d^4}{8\, I_x}\right) C_{\ell_p} \qquad [L^{-1}]$$

For the present study, A_3 can be considered constant. If we were given sufficient spin data (obtained, say, by measuring the position on each spark photograph of two distinguishable pins placed in the base of the missile), we could obtain the "best" values of p_o and A_3 by the Chapman-Kirk technique or by a fit of the data to the solution of (48):

$$p = p_o \exp\,[A_3\,(x - x_o)] \qquad (49)$$

In the present study, however, such spin data was unavailable. Yet the roll rate p was needed in the yaw equations. We resolved this problem by assuming that p is a known linear function of time:

$$p = p_o\,[1 + A_3\, V_o\,(t - t_o)] \qquad (50)$$

where $p_o = 2\,\pi\, V_o/(nd)$, rad/sec

n = rifling, cal/rev (see Table I)

and where A_3 was evaluated by assuming $C_{\ell_p} = -.013$ for all twelve rounds.

**In some texts (e.g., in References 13 and 14), the combination* pd/(2V) *is preferred to* pd/V *and in those texts the* C_{ℓ_p} *must be interpreted accordingly:*

$$\left(C_{\ell_p}\right)_{\frac{pd}{2V}} = 2\left(C_{\ell_p}\right)_{\frac{pd}{V}}$$

C. The Yaw Equations

The yaw equations describe the wobbling motion of the missile's longitudinal axis $\vec{A}$ about the tangent to the trajectory, that is, about the velocity vector $\vec{V}$. Any two variables sufficient to orient $\vec{V}$ with respect to $\vec{A}$ can serve as the dependent variables. In the usual BRL Transonic Range reduction, the dependent variables are the yaw components ξ_2 and ξ_3; here we use the Euler angles ψ and θ and the governing equations (45) and (46). We define the cross-moments acting on the missile as follows*:

$$M_2 + i M_3 = \left(\rho \frac{V^2}{2}\right)\left(\frac{\pi d^2}{4}\right)(d)\left\{C_{M_q}\left[\frac{(\dot\theta + i\dot\psi\cos\theta)\,d}{V}\right] + \left[\left(\frac{pd}{V}\right)C_{M_{p\alpha}} - i\,C_{M_\alpha}\right]\left(\frac{u_2 + i u_3}{V}\right)\right\}\quad [ML^2T^{-2}] \tag{51}$$

where**

$$\left.\begin{aligned} C_{M_\alpha} &= C_{M_{\alpha o}} + C_{M_{\alpha 2}}\,\delta^2 + C_{M_{\alpha 4}}\,\delta^4 \\ C_{M_{p\alpha}} &= C_{M_{p\alpha o}} + C_{M_{p\alpha 2}}\,\delta^2 + C_{M_{p\alpha 4}}\,\delta^4 \\ C_{M_q} &= C_{M_{qo}} + C_{M_{q2}}\,\delta^2 \end{aligned}\right\} \tag{52}$$

**Provision was made in the computer program to cope with an asymmetrical missile by including some trim terms not shown in (51) above. These terms were not used in the present study.*

***Note that* $C_{M_{p\alpha}}$ *in (51) is multiplied by the nondimensional spin* pd/V, *so that the remarks on* C_{ℓ_p} *in a previous footnote apply to* $C_{M_{p\alpha}}$ *as well. That is,*

$$\left(C_{M_{p\alpha}}\right)_{\frac{pd}{2V}} = 2\left(C_{M_{p\alpha}}\right)_{\frac{pd}{V}}$$

Likewise, C_{M_q} *as defined here is half the* C_{M_q} *of References 13 and 14.*

Substituting (51) in (45) and (46), we obtain the final form of the yaw equations:

$$\ddot{\theta} = A_2 \left[(V u_3)\, C_{M_\alpha} + (u_2\, p\, d)\, C_{M_{p\alpha}} + (d\, V\, \dot{\theta})\, C_{M_q} \right]$$

$$- \left[(I_x/I_y)\, p + \dot{\psi} \sin\theta \right] \dot{\psi} \cos\theta \qquad (53)$$

$$\ddot{\psi} = \left\{ A_2 \left[-(V u_2)\, C_{M_\alpha} + (u_3\, p\, d)\, C_{M_{p\alpha}} + (d\, V\, \dot{\psi} \cos\theta)\, C_{M_q} \right] \right.$$

$$\left. + \left[(I_x/I_y)\, p + 2\, \dot{\psi} \sin\theta \right] \dot{\theta} \right\} / \cos\theta \qquad (54)$$

where A_2 is a known constant:

$$A_2 = \frac{\pi\, \rho\, d^3}{8\, I_y} \qquad [L^{-2}]$$

Equations (53) and (54) involve four initial conditions, the eight aerodynamic parameters indicated in (52) and (possibly) one physical parameter, I_x/I_y. Thus a maximum of thirteen constants could be determined for each round by an application of the Chapman-Kirk technique to (53) and (54). However, for any computer run, any one or more of the eight aerodynamic constants in (52) could be treated as a known constant rather than as a parameter to be determined. For example, $C_{M_{q_2}}$ was fixed at zero (that is, ignored) in all the present computer runs.

Note that in addition to θ, ψ and their derivatives, (53) and (54) involve five dependent variables:

$$V,\ p,\ \delta^2,\ u_2 \text{ and } u_3$$

We have already given equations adequate for simulating V and p as functions of time, namely, (37) and (50), respectively. The squared yaw can be obtained by (17):

$$\delta^2 = (u_2^2 + u_3^2)\,/V^2 \qquad (17a)$$

provided we know u_2 and u_3. Thus the ability to solve (53) and (54) hinges now on our ability to generate u_2 and u_3. We proceed now to derive the required equations. Let

$$\vec{F} \equiv (F_1, F_2, F_3)_{FP}$$

$$= \text{the aerodynamic force vector}$$

and assume that the only forces acting on the missile are the aerodynamic and gravitational forces. Then the force equation is

$$\left.\begin{aligned} \vec{F} + \vec{G} &= m \left.\frac{d\vec{V}}{dt}\right|_I \\ &= m\,[(\dot{u}_1, \dot{u}_2, \dot{u}_3) + \vec{\omega}_{FP(R)} \times \vec{V}]_{FP} \end{aligned}\right\} \quad (55)$$

or, by (21b) and (22),

$$F_1 + mg \sin \theta = m\,(\dot{u}_1 - u_2\,\dot{\psi} \cos \theta + u_3\,\dot{\theta}) \quad (56)$$

$$F_2 = m\,(\dot{u}_2 + u_3\,\dot{\psi} \sin \theta + u_1\,\dot{\psi} \cos \theta) \quad (57)$$

$$F_3 - mg \cos \theta = m\,(\dot{u}_3 - u_1\,\dot{\theta} - u_2\,\dot{\psi} \sin \theta) \quad (58)$$

We assume that the fixed-plane components of the aerodynamic force are given by

$$\left.\begin{aligned} F_1 &= -\,m\,A_1\,C_D\,V\,u_1 \\ F_2 + i\,F_3 &= -\,m\,A_1 \left(C_{N_\alpha} V + i\,C_{N_{p\alpha}}\,pd \right) (u_2 + i\,u_3) \end{aligned}\right\} \quad (59)$$

where for the present purposes, C_{N_α} and $C_{N_{p\alpha}}$ are considered <u>known constants*</u>. Then equations (57) and (58) can be written as:

**As with* C_{ℓ_p}, $C_{M_{p\alpha}}$ *and* C_{M_q}, *we have*

$$\left(C_{N_{p\alpha}} \right)_{\frac{pd}{2V}} = 2 \left(C_{N_{p\alpha}} \right)_{\frac{pd}{V}}$$

$$\dot{u}_2 = - A_1 \left[(V u_2) C_{N_\alpha} - (p d u_3) C_{N_{p\alpha}} \right]$$

$$- (u_1 \cos \theta + u_3 \sin \theta) \dot{\psi} \tag{60}$$

$$\dot{u}_3 = - A_1 \left[(V u_3) C_{N_\alpha} + (p d u_2) C_{N_{p\alpha}} \right]$$

$$+ u_1 \dot{\theta} + u_2 \dot{\psi} \sin \theta - g \cos \theta \tag{61}$$

where

$$u_1 = (V^2 - u_2^2 - u_3^2)^{\frac{1}{2}} = (1 - \delta^2) V$$

Thus we generate u_2 and u_3 by assigning values to C_{N_α}, $C_{N_{p\alpha}}$, u_{20} and u_{30} and then solving (60) and (61) simultaneously with the yaw equations (53) and (54).

An alternative method for generating u_2 and u_3 follows at once from assumptions (23a) and (24a) that $\psi = \psi_M$ and $\theta = \theta_M$. By (16), (25) and (26) we have

$$u_2 = - V \sin \psi$$

$$u_3 = V \cos \psi \sin \theta$$

These expressions eliminate the need for (60) and (61). The only drawback is that the resulting computer program couldn't be used in future cases where assumptions (23a) and (24a) are invalid.

While we didn't use this simplification, we did ease the labor of computation by modifying the auxiliary set of yaw equations produced by parametric differentiation. That is, we replaced the functions f_j in equation (14) by approximating expressions. The validity of this procedure depends, of course, on the adequacy of the approximations.

One final remark: a by-product of the yaw reduction is δ^2 as a function of time. As mentioned in section IV A, a second drag reduction could be performed with this computed δ^2 as input. If the output of the second drag reduction differs significantly from the

original drag output, a second yaw reduction based on the new drag data should then be run. For the present study, none of this was necessary.

D. The CG Equations

The motion of the missile's CG along the trajectory is defined by the force equation:

$$\vec{F} + \vec{G} = m \left. \frac{d^2R}{dt^2} \right|_I \tag{62}$$

We are interested in the range components of (62). Whereas in (30) and (55) we could treat the range system as an inertial system, here such simplification is not as defensible. For the range (earth-fixed) system, the acceleration is given by

$$\left. \frac{d^2\vec{R}}{dt^2} \right|_I = [(\ddot{x},\ddot{y},\ddot{z}) + \vec{C} + \vec{C}_T]_R \tag{63}$$

where $\vec{C}_T$ is the centripetal acceleration, which we promptly ignore, and where $\vec{C}$ is the Coriolis acceleration, which we retain:

$$\vec{C} = 2\,\vec{\omega}_E \times \vec{V}$$

where

$$\vec{\omega}_E = \text{the earth's angular velocity vector}$$

$$= \omega_E\,(\cos\theta_L \cos\theta_A,\ \cos\theta_L \sin\theta_A,\ \sin\theta_L)_R$$

$$\omega_E = 2\,\pi \text{ radians/sidereal day}$$

$$\cong 0.00007292 \text{ rad/sec}$$

and where θ_A and θ_L are known constants defined in the List of Symbols. If we make the approximation

$$\vec{V} = (\dot{x},\dot{y},\dot{z})_R \cong (V,\ 0,\ 0)_R$$

then the Coriolis acceleration is given by

$$\vec{C} = 2\,\omega_E\,V\,(0,\ \sin\theta_L,\ -\cos\theta_L\,\sin\theta_A)_R \tag{64b}$$

Defining

$$\vec{F} = (F_X,\ F_Y,\ F_Z)_R = (F_1,\ F_2,\ F_3)_{FP}$$

and using (21a), (63) and (64b), we can write (62) in the form

$$\left.\begin{aligned}
\ddot{x} &= F_X/m \\
\ddot{y} &= F_Y/m - (2\,\omega_E \sin\theta_L)\,V \\
\ddot{z} &= F_Z/m + (2\,\omega_E \cos\theta_L \sin\theta_A)\,V - g
\end{aligned}\right\} \tag{65}$$

To obtain expressions for F_X, F_Y and F_Z, we assume that the fixed-plane components of $\vec{F}$ have the form already indicated in (59):

$$\left.\begin{aligned}
F_1 &= -\,m\,A_1\,C_D\,V\,u_1 \\
&= -\,m\,A_1\,C_{DA}\,V^2 \\
F_2 + i\,F_3 &= -\,m\,A_1\left(C_{N_\alpha}\,V + i\,C_{N_{p\alpha}}\,pd\right)(u_2 + i\,u_3)
\end{aligned}\right\} \tag{66}$$

where

$$C_{DA} = \left(\frac{u_1}{V}\right) C_D = (1 - \delta^2)\,C_D$$

$$= \text{axial drag coefficient}$$

For the yaw reduction, C_{N_α} and $C_{N_{p\alpha}}$ were considered known constants and F_1 wasn't needed; here we assume

$$C_{DA} = C_{DAo} + C_{DA_2}\,\delta^2$$

$$C_{N_\alpha} = C_{N_{\alpha o}} + C_{N_{\alpha 2}}\,\delta^2$$

$$C_{N_{p\alpha}} = C_{N_{p\alpha o}} + C_{N_{p\alpha 2}}\,\delta^2$$

where the six coefficients on the right-hand side are unknown constants to be determined by the Chapman-Kirk technique*. The range components of the aerodynamic force are obtained by multiplying the fixed-plane components by the transpose of the transformation matrix given in equation (20):

$$\left.\begin{aligned} F_X &= F_1 \cos\theta\cos\psi - F_2 \sin\psi + F_3 \sin\theta\cos\psi \\ F_Y &= F_1 \cos\theta\sin\psi + F_2 \cos\psi + F_3 \sin\theta\sin\psi \\ F_Z &= -F_1 \sin\theta + F_3 \cos\theta \end{aligned}\right\} \quad (67)$$

Substituting (66) and (67) in (65) we obtain the final form of the CG equations:

$$\begin{aligned} \ddot{x} = -A_1 \Bigg\{ & \left(V^2 C_{DA}\right) \cos\theta\cos\psi \\ & - \left[(V u_2)\, C_{N_\alpha} - (pd\, u_3)\, C_{N_{p\alpha}}\right] \sin\psi \\ & + \left[(V u_3)\, C_{N_\alpha} + (pd\, u_2)\, C_{N_{p\alpha}}\right] \sin\theta\cos\psi \Bigg\} \end{aligned} \quad (68)$$

Since $C_{DA} = (1 - \delta^2)\, C_D = C_{Do} + \left(C_{D_2} - C_{Do}\right)\delta^2 - C_{D_2}\,\delta^4$, where C_{Do} and C_{D_2} are well-determined from the drag reduction, we could, as an alternative, consider C_{DA} a known function of δ^2, thus reducing to four the number of aerodynamic coefficients to be determined by the CG equations.

$$\ddot{y} = -A_1 \left\{ \left(V^2 C_{DA} \right) \cos\theta \sin\psi + \left[(V u_2) C_{N_\alpha} - (pd\, u_3) C_{N_{p\alpha}} \right] \cos\psi + \left[(V u_3) C_{N_\alpha} + (pd\, u_2) C_{N_{p\alpha}} \right] \sin\theta \sin\psi \right\} - (2\,\omega_E \sin\theta_L)\, V \tag{69}$$

$$\ddot{z} = A_1 \left\{ \left(V^2 C_{DA} \right) \sin\theta - \left[(V u_3) C_{N_\alpha} + (pd\, u_2) C_{N_{p\alpha}} \right] \cos\theta \right\} + (2\,\omega_E \cos\theta_L \sin\theta_A)\, V - g \tag{70}$$

The right-hand sides of equations (68 - 70) involve six dependent variables:

$$V,\ p,\ u_2,\ u_3,\ \theta \text{ and } \psi.$$

The first of these can be generated internally:

$$V = (\dot{x}^2 + \dot{y}^2 + \dot{z}^2)^{1/2}$$

The other five must be brought in from outside. For the spin p, we used the linear expression given in (50). The variables u_2, u_3, θ and ψ were obtained from the yaw reduction, as described in the previous subsection.

The aerodynamic coefficients and initial conditions of equations (68 - 70) were adjusted by the Chapman-Kirk technique until the solution of the equations was a least squares fit to the measured $(x,y,z)_R$ values of the CG.

V. RESULTS

The results obtained in the present study can be interpreted better in the light of a little background on the four shell types.

A. Background

The 90mm M71 HE shell was designed to be fired from a 1/32 twist gun at Mach 2.4; at lower muzzle velocities, the shell was known to perform unsatisfactorily. The two rounds studied here (round 2-4203 at 7° average yaw and round 2-4204 at 5°) were fired at about Mach 0.93. At that muzzle velocity and a 1/32 twist, the gyroscopic stability factor s_g was slightly less than unity at launch, so that the shell was gyroscopically unstable when it emerged from the tube. However, the situation (like the shell) took some turns for the better. As the shell travelled downrange, (a) the velocity decay exceeded the spin decay and (b) the static moment coefficient decreased with decreasing Mach number. As a result, s_g increased, eventually crossing the value 1.0, so that the shell became and remained gyroscopically stable.

The M71 rounds were fired and initially reduced in 1956, but the values obtained for some of the coefficients were known to be nonsense. The photographic plates were reread several times in the next four years in a largely unsuccessful attempt to improve the linear theory fit. One positive result of all these readings is that the probable errors of the fit in the present study are smaller for the two M71 rounds than for any of the other rounds.

Before applying the Chapman-Kirk technique to the two M71 rounds, we simulated their motion on an analog computer, using partially linearized equations of motion. This side investigation served two purposes: it satisfied our curiosity as to the behavior of a shell flying along the borderline of gyroscopic stability - instability and it provided accurate estimates of the initial conditions required by the Chapman-Kirk technique. For the other three shell types, however, we were satisfied to use a simple digital subprogram to obtain the needed first estimates.

The M329A1 shell without extension was a 4.2-inch (107mm) spin-stabilized mortar shell to which a subcaliber cylindrica after-body - a boom - had been attached at the base (see Figure 1). This boom, about 0.35 caliber in diameter and 0.7 caliber long, was used to carry the ignition and propelling charge and to provide needed volume. The boom served no aerodynamic purpose and it was hoped at the time that the boom would have negligible aerodynamic effect on the shell's performance. Careful experiments[11] proved otherwise.

The M329A1 shell with extension carried, as its name might imply, an extension to the original boom, bringing the total boom length to 1.35 calibers (see Figure 1). Tests[11] comparing the M329A1 with and without the extension clearly indicated that the added length affected the aerodynamics, decreasing the stability and increasing the drag and the shell's tendency to fly erratically now and then.

The M329A1E1 shell differed from the two previous types in that it had a longer ogive, a shorter body, a boattail rather than a square base and a shorter over-all length (see Figure 1). The boom was about 0.31 caliber in diameter and 0.75 caliber long.

B. Drag Results

A comparison of the BRL drag results with those of the GE Chapman-Kirk approach is given in Table II. The last column lists the BRL values of C_{D_o} and C_{D_2} for each of the four projectile types, obtained by a least squares fit of the data for each round of the given type to the equation

$$C_{DR} = C_{D_o} + C_{D_2} (\overline{\delta^2})_{BRL} \qquad (71)$$

Although additional rounds of each of the four shell types were available for fitting BRL data to equation (71), it was considered a fairer comparison to work solely with the twelve given rounds. As a result, the fits for the M71 and M329A1E1 - where there are only two rounds each - are exact and unreliable. To make matters worse, the yaw levels of the two M71 rounds are roughly the same, so that we can expect the slope to be poorly determined. In fact, only for type M329A1 with extension, where there are five rounds (albeit only three yaw levels), can we regard the results of the fit with any semblance of confidence. In spite of all this, the BRL fitted values of C_{D_o} and C_{D_2} and the GE Chapman-Kirk values are not wildly dissimilar - they are, so to speak, in the same ball park - and perhaps that is all we can expect from such a meager number of rounds.

One encouraging feature of the GE data is that the values of C_{D_o} and C_{D_2} obtained are approximately the same for each round of a given type. This seems a necessary if not sufficient condition for trusting the results. For each round, we can also compare $\overline{C}_D$ (from equation 35) with the BRL range value C_{DR} shown in the next to last column of Table II. Here we find that the agreement is quite good when the yaw is small and poorer when the yaw is large.

C. Yaw Results

The yaw results are shown in Tables III - VI. The BRL values of C_{M_α}, C_{M_q} and $C_{M_{p\alpha}}$ shown for each round are - for the small yaw rounds - the values obtained by the standard BRL epicycle yaw reduction[1,2]. For the large yaw rounds (8615, 8618, 8621 and 8721), the BRL reduction values were corrected to compensate for the fact that certain geometrical terms in the yaw reduction - terms which are only significant for large yaw angles - are ignored in the BRL reduction.

Again it should be noted that all the coefficients C_{M_q} and $C_{M_{p\alpha}}$ shown in these tables, both BRL and GE values, are defined (see equation 51) as half the coefficients designated by the same symbols in, for example, References 9, 13 and 14. This disparity is unfortunate but necessary if the present paper is to be consistent with other BRL reports.

For most of the rounds, more than one GE case per round is shown in the tables. These cases differ in the selection of which parameters were fixed (indicated by an asterisk) and which were allowed to seek out their optimum values. As might be expected, the more parameters fitted, the smaller (with a few exceptions) was the probable error of the fit. For three rounds (8713, 8716 and 8981), GE assumed in one case per round that all the moment coefficients were constant. This affords a direct comparison with the BRL values.

The last row in Tables III - VI lists the four coefficients obtained by fitting the BRL data for each round of the given type to the equations

$$\left.\begin{aligned} \left(C_{M_\alpha}\right)_{BRL} &= C_{M_{\alpha o}} + C_{M_{\alpha_2}} \delta_e^2 \\ \left(C_{M_{p\alpha}}\right)_{BRL} &= C_{M_{p\alpha o}} + C_{M_{p\alpha_2}} \delta_e^2 \end{aligned}\right\} \quad (72)$$

where the constant δ_e^2 is an effective squared yaw, obtained as a by-product of the BRL yaw reduction. The previous remarks on the fit to Equation (71) apply with equal force to Equation (72): the results of the fit are not too trustworthy but are least suspect for the M329A1 with extension (Table IV).

Round 8618 and to a lesser extent round 8621 (Table IV) revealed a very strong Magnus moment nonlinearity. In fact, by a separate computer curve fit program, we determined that the expansion

$$C_{M_{p\alpha}} = C_{M_{p\alpha o}} + C_{M_{p\alpha_2}} \delta^2 + C_{M_{p\alpha_4}} \delta^4 + C_{M_{p\alpha_6}} \delta^6 + C_{M_{p\alpha_8}} \delta^8 \qquad (73)$$

was needed in these two rounds to obtain a reasonable fit to the given data. Accordingly, our main program was modified to allow for the last terms in (73) - terms that are missing in equation (52). Unfortunately, we were unable to determine values for the two new coefficients; the process diverged on every attempt. That is, it was not possible with the present technique and/or the available data to determine more than three terms of the Magnus moment coefficient expansion.

This situation should be investigated further. A first step would be to analyze by the present technique the output of a six-degrees-of-freedom program that simulates the motion of a projectile with a highly nonlinear Magnus moment. In this way, the quantity and the quality of the experimental data needed to obtain a satisfactory fit for this type of nonlinearity could be determined.

D. CG Results

The CG equations (68-70) were applied only to the two M329A1E1 rounds (see Table VII). The values obtained for the normal force coefficient are in good agreement with each other and with the BRL values. The Magnus force coefficient (which is less by that ubiquitous factor of two than the coefficient labelled $C_{Y_{p\alpha}}$ in Reference 14) is not very well determined. The BRL values of the axial drag coefficient shown in Table VII were obtained by the approximation

$$\left(C_{DA}\right)_{BRL} = (1 - \overline{\delta^2})\, C_{DR}$$

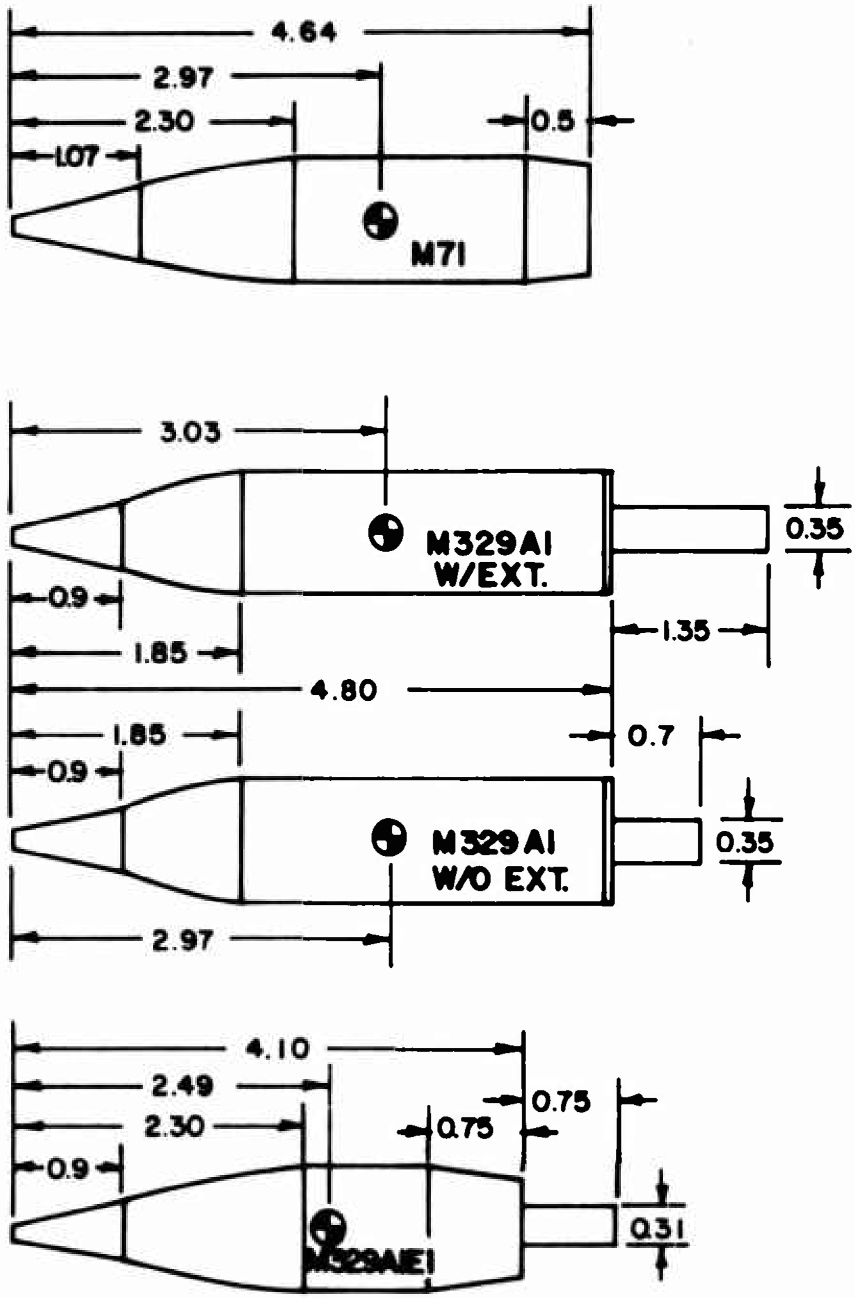
using the C_{DR} values in Table III.

VI. CONCLUSIONS AND RECOMMENDATIONS

We have shown that for twelve problem rounds the Chapman-Kirk technique could be applied satisfactorily to free-flight, enclosed-range data. However, before we can safely apply the technique on a steady, production basis to high-yaw rounds, additional theoretical study is needed. A systematic investigation should be made, whereby a variety of trajectories are computer-generated, suitable noise is introduced and the resultant data fed to the Chapman-Kirk system.

Some force and moment coefficients are more easily determined than others; for example, those depending mainly on the frequencies of the motion are more easily pinpointed than those related to the damping. For a given force or moment expansion in δ^2, the coefficients naturally become less reliable as the order of the term increases and shortly a limit is reached; when any term of order higher than this limit is included in the analysis, the process fails to converge (or, worse yet, converges to wrong answers). This limit is a function of the number and accuracy of the observations, so that any improvement in these factors (up to some point of diminishing return) would be helpful.

It might be possible to determine high order terms more accurately for a given set of data if the lowest order terms are fixed at good values. In particular, low yaw rounds (say, $\overline{\alpha_T} < 3°$) should establish adequate zero-yaw coefficients for use with the high yaw rounds. In general, the fewer the number of coefficients to be determined, the less likely that computational instabilities will arise.



NOTE: ALL DIMENSIONS ARE IN CALIBERS

FIGURE I SIMPLIFIED SKETCH OF THE FOUR PROJECTILE TYPES

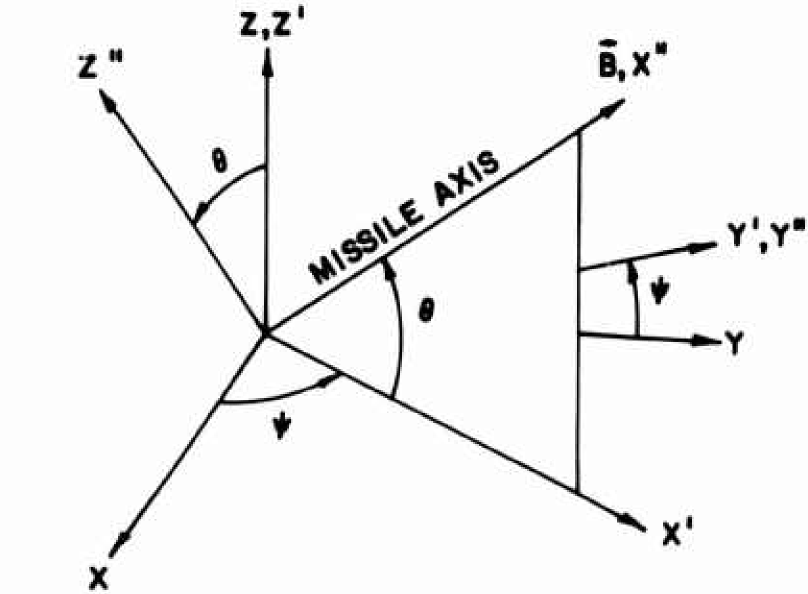


FIG. 2a ψ IS POSITIVE FOR A CCW ROTATION OF THE XY-PLANE ABOUT Z (THUS ψ AS SHOWN IN THE FIGURE IS POSITIVE)

θ IS POSITIVE FOR A CCW ROTATION OF THE Z'X'-PLANE ABOUT Y' (THUS θ AS SHOWN IN THE FIGURE IS NEGATIVE)

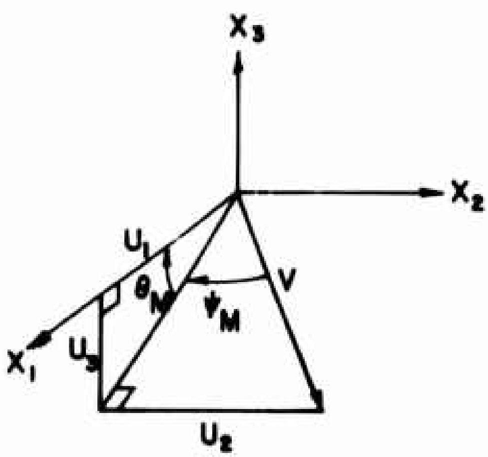


FIG. 2b

$$\psi_M = \sin^{-1}\left(\frac{-U_2}{V}\right)$$

$$\theta_M = \tan^{-1}\left(\frac{U_3}{U_1}\right)$$

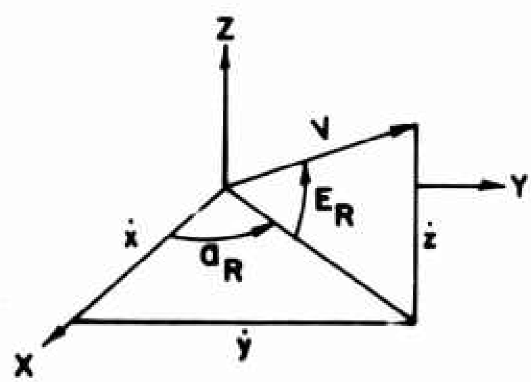


FIG. 2C

$$E_R = \sin^{-1}\left(\frac{\dot{z}}{V}\right)$$

$$a_R = \tan^{-1}\left(\frac{\dot{y}}{\dot{x}}\right)$$

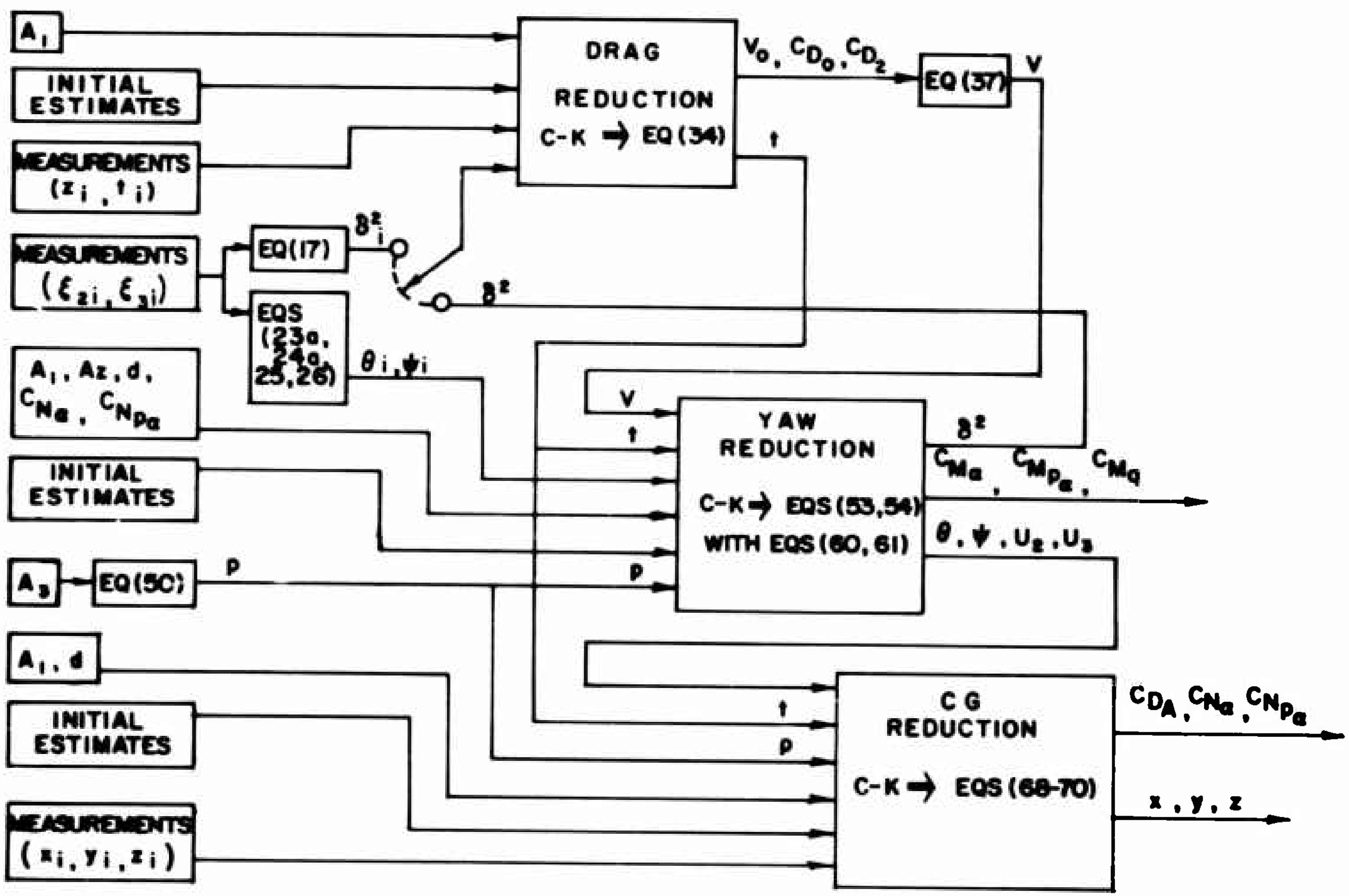


FIGURE 3 SIMPLIFIED BLOCK DIAGRAM OF THE DATA REDUCTION PROCEDURE

Table I. Physical Parameters and Environment

Type	BRL RD No. 2-	PHYSICAL PARAMETERS d [M]	m [KG]	I_x [KG-M^2]	I_y [KG-M^2]	CG (cal. from nose)	n=rifling [cal/rev]	ENVIRONMENT ρ [KG/M^3]	Vel. Sound [M/Sec]
M71	4203	.08926	10.614	.011720	.11067	2.974	32	1.1770	348.1
	4204	.08926	10.623	.011720	.11067	2.962	32	1.1722	348.7
M329A1 w/ext	8615	.10643	11.898	.018227	.23735	3.029	18	1.2043	345.4
	8617	.10645	11.898	.018215	.23884	3.034	18	1.1856	347.9
	8618	.10638	11.902	.018165	.23755	3.040	18	1.1825	348.2
	8621	.10643	11.902	.018211	.23818	3.032	18	1.1870	347.0
	8704	.10643	11.898	.018781	.24871	2.989	18	1.2046	344.7
M329A1 wo/ext	8713	.10643	11.793	.018248	.22698	2.997	18	1.1800	345.0
	8716	.10643	11.793	.018673	.23774	2.951	18	1.1814	344.9
	8721	.10643	11.793	.018673	.23774	2.961	18	1.1981	344.5
M329A1 E1	8981	.10640	10.006	.015080	.13025	2.487	20	1.1540	350.6
	8988	.10640	10.002	.015080	.13025	2.487	20	1.1527	351.2

Preceding page blank

Table II. Drag Reduction Results

Type	BRL RD 2- ($\overline{\alpha}_T$, deg)	GE DRAG REDUCTION RESULTS: PE of Fit, μ-sec (No. Sta.)	GE: Values at Station 1: t_o, sec (PE, μ-sec)	GE: Values at Station 1: V_o, m/sec (PE, m/sec)	GE: C_{D_o} (PE)	GE: C_{D_2} (PE)	GE: $\overline{C}_D$ (Eq 35)	BRL: C_{DR} (Eq 36)	BRL: (C_{D_o}, C_{D_2}) (Eq 71)
M71	4203 (6.8)	2.0 (14)	- .01932 (1.2)	325.5 (.003)	.162 (.002)	4.19 (.15)	.221	.219	(.185, 2.53)
	4204 (4.9)	2.8 (13)	- .01921 (2.2)	327.6 (.006)	.160 (.002)	5.27 (.23)	.199	.204	
M329A1 w/ext	8615 (14.2)	3.9 (14)	- .01838 (3.4)	221.53 (.0037)	.207 (.0003)	3.00	.392	.372	(.190, 3.59)
	8617 (5.3)	4.7 (10)	- .02615 (6.2)	219.54 (.0064)	.212 (.0006)	3.25*	.240	.237	
	8618 (14.6)	6.6 (11)	- .01759 (8.4)	214.57 (.0079)	.204 (.0008)	3.20*	.412	.389	
	8621 (24.6)	10.4 (9)	.00006 (12)	214.23 (.0192)	.197 (.011)	3.16 (.06)	.779	.815	
	8704 (5.9)	7.2 (14)	- .02750 (10)	216.83 (.0055)	.207 (.0005)	3.15*	.241	.238	
M329A1 wo/ext	8713 (10.2)	3.4 (12)	- .02716 (3.7)	216.62 (.01)	.220 (.006)	3.40 (.20)	.328	.323	(.223, 2.79)
	8716 (4.8)	11.3 (12)	- .02767 (1.6)	213.80 (.03)	.209 (.015)	3.31 (2.28)	.232	.231	
	8721 (18.3)	6.8 (12)	- .02780 (4.9)	213.84 (.006)	.215*	3.00 (.005)	.522	.479	
M329A1 E1	8981 (9.2)	4.3 (13)	- .02988 (5.2)	192.05 (.0107)	.129 (.008)	2.0 (.35)	.180	.174	(.115, 2.48)
	8988 (4.5)	6.0 (14)	- .02408 (4.2)	225.66 (.0061)	.117 (.0004)	2.0*	.129	.129	

*Held Constant

Table III. M71 Yaw Reduction Results

BRL RD 2-( )	Method (PE of Fit)	$C_{M_{\alpha o}}$ (PE)	$C_{M_{\alpha 2}}$ (PE)	C_{M_q} (PE)	$C_{M_{p\alpha o}}$ (PE)	$C_{M_{p\alpha 2}}$ (PE)	I_x/I_y (PE)
4203 (6.8°) (24 Sta)	BRL (.09°)	4.21		- 26.1	1.66		.1059*
	GE (.113°)	4.62 (.004)	- 4.8 (.21)	- 22 (2.0)	1.36 (.12)		.1059*
	GE (.096°)	4.61 (.003)	- 4.7 (.18)	- 8.8 (4.3)	0.21 (.35)	20.6 (6.1)	.1059*
	GE (.082°)	4.38 (.03)	- 1.12 (.56)	- 8.4 (4.1)	0.20 (.34)	19.9 (6.0)	.1041 (.0003)
4204 (4.9°) (19 Sta)	BRL (.08°)	4.23		- 44.2	2.52		.1059*
	GE (.118°)	4.58 (.001)	- 5.0*	- 28 (2.8)	3.20 (.16)		.1059*
	GE (.083°)	4.62 (.005)	- 8.3 (.46)	- 5.9 (4.2)	-0.43 (.33)	99.4 (9.3)	.1059*
	GE (.064°)	4.37 (.03)	- 2.41 (.90)	- 6.8 (3.6)	-0.52 (.29)	115.5 (8.2)	.1039 (.0003)
Combined	BRL Eq. (72)	4.25	- 0.83		3.23	- 35.6	

*Held Constant

Table IV. M329A1 (w/ext) Yaw Reduction Results

BRL RD 2-( )	Method (PE of Fit)	$C_{M_{\alpha o}}$ (PE)	$C_{M_{\alpha 2}}$ (PE)	$C_{M_{\alpha 4}}$ (PE)	C_{M_q} (PE)	$C_{M_{p\alpha o}}$ (PE)	$C_{M_{p\alpha 1}}$ (PE)	$C_{M_{p\alpha 2}}$ (PE)	$C_{M_{p\alpha 4}}$ (PE)
8615 (14.2°)	BRL (.227°)	3.18			- 12.7	1.15			
	GE (.122°)	4.46 (.08)	- 9.5 (.77)		- 0.35 (3.25)	- 1.12 (.57)		22.0 (5.28)	
8617 (5.3°)	BRL (.135°)	4.32			- 3.2	.083			
	GE (.098°)	4.31 (.009)			- 5.5 (1.3)	1.37 (.60)		- 100 (48)	
	GE (.093°)	4.64 (.09)	- 24.7 (7.1)		- 6.1 (1.2)	1.35 (.54)		- 95 (48)	
8618 (14.6°)	BRL (.251°)	3.15			- 11.6	1.09			
	GE (.162°)	4.40 (.11)	- 8.9 (.93)		- 12.6 (.89)	1.23 (.06)			
	GE (.129°)	4.43 (.09)	- 9.0 (.77)		- 1.22 (3.43)	- .83 (.62)		17.8 (5.4)	
	GE (.125°)	4.36 (.08)	- 8.4 (.73)		2.47 (3.92)	- 2.83 (1.03)	11.5 (2.96)		
	GE (.115°)	4.30 (.08)	- 7.9 (.69)		2.98 (3.34)	- 2.33 (.77)		53.0 (13.7)	- 200 (80)
	GE (.097°)	4.68 (.10)	- 17.8 (1.9)	66.4 (12.2)	3.66 (2.68)	- 2.64 (.63)		59.5 (11.4)	- 258 (66)

(continued)

Table IV. M329A1 (w/ext) Yaw Reduction Results (continued)

BRL RD 2-( )	(PE of Fit)	$C_{M_{\alpha o}}$ (PE)	$C_{M_{\alpha 2}}$ (PE)	$C_{M_{\alpha 4}}$ (PE)	C_{M_q} (PE)	$C_{M_{p\alpha o}}$ (PE)	$C_{M_{p\alpha 1}}$ (PE)	$C_{M_{p\alpha 2}}$ (PE)	$C_{M_{p\alpha 4}}$ (PE)
8621 (24.6°)	BRL (1.40°)	1.75			- 25.4	2.61			
	GE (.275°)	3.85*	- 6.1 (.38	8.6 (1.5)	- 5.75*	- 0.7 (.42)		20.2 (5.0)	- 29.6 (13.9)
	GE (.238°)	3.85 (.18)	- 6.7 (1.5)	11.3 (3.6)	0.9 (2.9)	- 2.4 (.87)		33.4 (7.8)	- 63.1 (20.3)
8704 (5.9°)	BRL (.135°)	4.16			- 5.85	0.10			
	GE (.097°)	4.[illegible]2 (.06)	- 16.1 (4.0)		- 5.75 (.87)	0.92 (.36)		- 55 (24.3)	
Combined	BRL Eq. (72)	4.11	- 6.0			0.192		6.09	

*H. ld Constant

Table V. M329A1 (wo/ext) Yaw Reduction Results

BRL RD 2-( )	Method (PE of Fit)	$C_{M_{\alpha o}}$ (PE)	$C_{M_{\alpha_2}}$ (PE)	$C_{M_{\alpha_4}}$ (PE)	C_{M_q} (PE)	$C_{M_{p\alpha o}}$ (PE)	$C_{M_{p\alpha_2}}$ (PE)	$C_{M_{p\alpha_4}}$ (PE)
8713 (10.2°)	BRL (.11°)	3.44			- 4.23	.065		
	GE (.096°)	3.38 (.005)			- 4.35 (.47)	.085 (.03)		
	GE (.094°)	3.28 (.04)	2.21 (.83)		- 4.15 (.50)	- .05 (.22)	2.78 (4.81)	
	GE (.093°)	3.28 (.04)	2.22 (.83)		- 4.25 (.45)	.08 (.03)		
8716 (4.8°)	BRL (.07°)	3.49			- 4.08	- .058		
	GE (.071°)	3.28 (.044)	4.52 (4.42)		- 3.58 (.70)	- .08 (.04)		
	GE (.070°)	3.32 (.006)			- 3.63 (.69)	- .075 (.03)		
	GE (.069°)	3.32 (.006)			- 3.45 (.73)	.195 (.24)	- 27.3 (24.7)	
8721 (18.3°)	BRL (.33°)	2.83			- 5.54	.36		
	GE (.135°)	3.09 (.05)	- .67 (.28)		- 2.55 (.88)	- .55 (.24)	6.25 (1.5)	
	GE (.110°)	3.30*	- 4.1 (.27)	13.6 (1.7)	- 3.75*	.025*	- 2.0 (1.1)	29.8 (7.4)
	GE (.103°)	3.67 (.08)	- 8.6 (1.0)	27.8 (3.6)	- 3.85 (1.1)	.2 (.45)	- 2.8 (6.2)	71.8 (22.2)
Combined	BRL Eq. (72)	3.59	- 4.54			- .075	2.69	

*Held Constant

Table VI. M329A1E1 Yaw Reduction Results

BRL RD 2-( )	Method (PE of fit)	$C_{M_{\alpha o}}$ (PE)	$C_{M_{\alpha_2}}$ (PE)	C_{M_q} (PE)	$C_{M_{p\alpha o}}$ (PE)	$C_{M_{p\alpha_2}}$ (PE)	$C_{M_{p\alpha_4}}$ (PE)
8981 (9.2°)	BRL (.104°)	2.975		- 2.62	- .180		
	GE (.100°)	2.988 (.004)		- 2.86 (.31)	- .175 (.02)		
	GE (.099°)	3.07 (.04)	- [illegible] (1.3)	- 2.73 (.47)	- .05 (.26)	- 3.58 (8.0)	
	GE (.092°)	3.12*	- 3.83 (0.1)	- 2.31 (.31)	.5*	- 40.3 (7.2)	511 (183)
	GE (.087°)	3.06 (.04)	- 2.08 (1.1)	- 1.24 (.74)	1.8 (.70)	- 114 (41)	1406 (515)
8988 (4.5°)	BRL (.093°)	3.143		- 3.06	.14		
	GE (.105°)	3.145 (.009)		- 2.84 (.75)	.405 (.44)	- 29.9 (52)	
	GE (.105°)	3.014 (.07)	15.5 (8.3)	- 2.76 (.74)	.376 (.44)	- 27.2 (52)	
Combined	BRL (Eq. 72)	3.20	- 6.79		.250	- 12.9	

*Held Constant

Table VII. M329A1E1 CG Reduction Results

BRL RD 2-( )	Method (PE of Fit)	$C_{N_{\alpha o}}$ (PE)	$C_{N_{\alpha_2}}$ (PE)	$C_{N_{p\alpha o}}$ (PE)	$C_{N_{p\alpha_2}}$ (PE)	C_{DA_o} (PE)	C_{DA_2} (PE)
8981 (9.2°)	BRL (.0019)	1.403		.0566		.170	
	GE (.0023)	1.396 (.012)		- .5*	14.7 (2.4)	.117*	1.08*
	GE (.0021)	1.412 (.013)		.0435 (.075)		.117 (.013)	1.08 (.54)
	GE (.0020)	1.402 (.011)		.0585 (.072)		.143 (.0005)	
	GE (.0017)	1.364 (.068)	2.88 (2.2)	1.27 (.41)	- 34.8 (12)	.115 (.012)	1.10 (.58)
	GE (.0017)	1.444 (.010)		1.04 (.37)	- 29.0 (11)	.117*	1.08*
8988 (4.5°)	BRL (.0019)	1.402		- .64		.128	
	GE (.0023)	1.400 (.026)		- .555 (.17)		.122 (.009)	- .295 (1.53)
	GE (.0023)	1.401 (.026)		- .555 (.17)		.120 (.0005)	
	GE (.0022)	1.315 (.033)		- 2.655 (1.14)	228 (123)	.121 (.0005)	

*Held Constant

REFERENCES

1. C. H. Murphy, "Data Reduction for the Free Flight Spark Ranges," Ballistic Research Laboratories Report No. 900, 1954, AD 35833.

2. C. H. Murphy, "Free Flight Motion of Symmetric Missiles," Ballistic Research Laboratories Report 1216, 1963, AD 442757.

3. H. F. Meissinger, "The Use of Parameter Influence Coefficients in Computer Analysis of Dynamic Systems," Proc. Western Joint Computer Conference, San Francisco, California, May 1960, pp. 181-192.

4. G. A. Bekey and W. J. Karplus, *Hybrid Computation*, John Wiley & Sons, Ind., New York, 1968.

5. T. R. Goodman, "System Identification and Prediction - An Algorithm Using a Newtonian Iteration Procedure," *Quarterly of Applied Mathematics*, 24, No. 3, 1966, pp. 249-255.

6. T. R. Goodman and T. P. Sargent, "A Method for Identifying Nonlinear Systems with Applications to Vehicle Dynamics and Chemical Kinetics," Oceanics, Inc., Technical Report No. 71-83, 1971.

7. J. W. Bradley, "CHLOE: a FORTRAN Subroutine for Fitting Ordinary Differential Equations to Observed Data," Ballistic Research Laboratories Memorandum Report 2184, 1972, AD 743878.

8. G. T. Chapman and D. B. Kirk, "A New Method for Extracting Aerodynamic Coefficients from Free-Flight Data," *AIAA Journal*, 8, No. 4, 1970, pp. 753-758.

9. R. H. Whyte and J. G. Beliveau, "Non-Linear Free Flight Aerodynamic Reduction Using Angle of Attack or Angular Rate Data," General Electric Report No. GE T1SR69APB6, Armament Department, Burlington, Vermont, 1969.

10. D. R. Cormier, "Numerical Integration Method for Computing Stability Coefficients," Department of Aerospace and Mechanical Engineering, University of Notre Dame, Notre Dame, Indiana, 1969.

11. L. C. MacAllister, V. Oskay, B. J. Reiter, R. Klein and W. Gazdayka, "The Effect of a Subcaliber Cylindrical After-Body on the Behavior of Spin-Stabilized Projectiles," AIAA Paper No. 70-558, AIAA Atmosphere Flight Mechanics Conference, Tullahoma, Tennessee, 1970.

12. T. E. Bullock, M. H. Clarkson and D. C. Daniel, "Preliminary Report On Extracting Aerodynamic Coefficients From Dynamic Data," University of Florida Technical Report AFATL-TR-72-52 under contract with the Air Force Armament Laboratory (DLGC), Eglin Air Force Base, Florida, 1972.

REFERENCES (condinued)

13. R. H. Whyte and W. H. Mermagen, "A Method for Obtaining Aerodynamic Coefficients from Yawsonde and Radar Data," AIAA Paper No. 72-978, AIAA 2nd Atmospheric Flight Mechanics Conference, Palo Alto, California, 1972.

14. R. H. Whyte and A. Jeung, "Aerodynamic Reduction of Free Flight Transonic Range Data Utilizing Numerical Integration," General Electric Report No. GE 71APB514, Armament Department, Burlington, Vermont, 1971.

15. W. K. Rogers, Jr., "The Transonic Free Flight Range," Ballistic Research Laboratories Report 1044, 1958, AD 200177.

16. E. D. Boyer, "Aerodynamic Properties of the 90mm HE M-71 Shell," Ballistic Research Laboratories Memorandum Report 1475, 1963, AD 411804.

17. C. E. Knadler, Jr., "A Method for Determining the Parameters of Ordinary Differential Equations," NOL TR 68-1972, 1968.

18. D. W. Marquardt, "An Algorithm for Least-Squares Estimation of Nonlinear Parameters," *J. Soc. Indust. Appl. Math.*, 11, No. 2, June 1963, pp. 431-441.

19. P. R. Bevington, *Data Reduction and Error Analysis for the Physical Sciences*, McGraw-Hill Book Co., New York, 1969.